

The Time Compression Paradox: A Unified Theory of Why Artificial Intelligence Increases Rather Than Eliminates Work

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Abstract

We present a unified theory of the **Time Compression Paradox** (TCP): the counter-intuitive observation that artificial intelligence, by compressing the time required for cognitive tasks, *increases* rather than eliminates total work. This paper synthesizes three interacting mechanisms into a single dynamical framework. *Mechanism I: The Jevons Paradox of Intelligence* establishes that elastic demand for cognitive labor ($\varepsilon > 1$) ensures that cheaper cognition increases total cognitive expenditure [Long, 2026a]. *Mechanism II: Opportunity Space Expansion* demonstrates that reduced task costs render previously infeasible tasks viable, with the frontier expanding superlinearly ($\beta > 1$) in the compression ratio [Long, 2026b]. *Mechanism III: Competitive Dynamics* shows that market competition forces all agents to exploit the expanded frontier, preventing the time surplus from being consumed as leisure [Long, 2026c]. We prove that these three mechanisms interact through positive feedback loops that preclude any finite steady state: total cognitive work $W(\alpha)$ is monotonically increasing in AI capability α , with a work multiplier bounded below by $\rho^{\beta-1}$. A calibrated dynamical system model with five state variables projects a 10–100× increase in total cognitive work within a generation. We provide the definitive mathematical statement of the TCP axiom, complete proofs of the monotonicity and no-equilibrium theorems, a 30-year numerical simulation under three scenarios, and empirical evidence from software development, content creation, and scientific research. The production constraint shifts from execution time to imagination bandwidth, and the AI age will be characterized not by leisure but by unprecedented cognitive industry.

Keywords: Time Compression Paradox, Jevons Paradox, cognitive automation, opportunity space expansion, competitive dynamics, induced demand, endogenous growth, labor economics, dynamical systems, AI economics **Paper Series:** This is Paper 4 of 4 in the Time Compression Paradox series. Companion papers: Long [2026a] (Mechanism I), Long [2026b] (Mechanism II), Long [2026c] (Mechanism III).

1 Introduction

The dominant public narrative surrounding artificial intelligence and labor markets is one of *displacement*: AI will automate tasks, rendering human workers unnecessary, leading to mass unemployment and economic dislocation [Frey and Osborne, 2017, Acemoglu and Restrepo, 2020]. While this framing captures a real transitional dynamic, it fundamentally

mischaracterizes the steady-state outcome. The historical record of technological revolutions reveals a strikingly different pattern: productivity-enhancing technologies *increase* total work rather than decrease it.

This paper presents the definitive formulation of the **Time Compression Paradox** (TCP), a unified theory explaining why AI increases work. We formalize the following axiom:

Axiom 1 (Time Compression Paradox). *Artificial intelligence does not eliminate work; it compresses the amount of work achievable within a given time period, which paradoxically creates more work.*

This axiom, while initially counter-intuitive, follows directly from the interaction of three well-established economic phenomena that we have analyzed independently in companion papers:

- (i) **The Jevons Paradox of Intelligence** [Long, 2026a]: When the efficiency of cognitive labor increases, total consumption of cognitive labor increases rather than decreases, provided demand elasticity exceeds unity.
- (ii) **Opportunity Space Expansion** [Long, 2026b]: Reducing the cost of cognitive tasks renders previously infeasible tasks viable, superlinearly expanding the frontier of possible work.
- (iii) **Competitive Dynamics** [Long, 2026c]: Market competition forces all agents to exploit the expanded frontier, preventing the time surplus from being consumed as leisure.

The contribution of this paper is to synthesize these three mechanisms into a unified dynamical system, prove that their interaction produces monotonically increasing total work, and demonstrate through calibrated simulation that the resulting expansion will be the largest in the history of cognitive labor.

1.1 The Displacement Narrative and Its Limits

The displacement narrative has a long pedigree. From the Luddites of the early 19th century to Keynes’s prediction of 15-hour work weeks [Keynes, 1930], each generation has anticipated that technological progress would reduce the need for human labor. In every case, the prediction has been falsified—not because the technology failed to deliver its promised efficiency, but because efficiency gains expanded the scope of human ambition.

The AI displacement narrative [Frey and Osborne, 2017, Brynjolfsson and McAfee, 2014, Agrawal et al., 2019] is the latest instantiation of this pattern. It correctly identifies that AI will automate specific

cognitive tasks. Where it errs is in assuming a fixed work frontier—that the set of tasks to be done is static, so that automating some fraction reduces total work by that fraction. The TCP framework shows that the work frontier is endogenous to productivity, and that the frontier expansion systematically outpaces task automation.

1.2 Structure of the Paper

This paper is organized as follows. Section 2 states the TCP formally and surveys historical precedents. Section 3 develops the formal model: definitions, the work generation function, and core assumptions. Sections 4–6 summarize the three mechanisms from the companion papers. Section 7 presents new analysis of how the mechanisms interact. Section 8 assembles the full dynamical system. Section 9 proves the monotonicity and no-equilibrium theorems. Section 10 analyzes the constraint shift from production to imagination. Section 11 contrasts the industrial and AI ages. Section 12 presents the complete numerical analysis. Section 13 examines empirical evidence. Section 14 discusses the sociology of compressed time. Section 15 outlines extensions and future directions. Section 16 concludes.

2 The Time Compression Paradox: Statement and Context

2.1 The Formal Axiom

We begin with the definitive mathematical statement of the TCP.

Axiom 2 (TCP, Formal Version). *Let $W(\alpha)$ denote total cognitive work as a function of AI capability level $\alpha \geq 0$, and let $\rho(\alpha) = T(0)/T(\alpha) \geq 1$ be the compression ratio. Then:*

$$W(\alpha) = W(0) \cdot \rho(\alpha)^{\beta-1} \quad (1)$$

where $\beta > 1$ is the opportunity elasticity. The exponent $\beta - 1$ reflects the net effect: the opportunity space grows as ρ^β (Assumption 2), while work per task is compressed by a factor of $1/\rho$. Consequently:

$$\frac{dW}{d\alpha} = W(0) \cdot (\beta - 1) \cdot \rho(\alpha)^{\beta-2} \cdot \frac{d\rho}{d\alpha} > 0 \quad (2)$$

That is, total cognitive work is strictly increasing in AI capability.

The axiom has a simple verbal form: *more productivity creates more possibilities, more possibilities create more projects, and more projects create more work.* The mathematical content is that the rate of work creation (via frontier expansion) exceeds the rate of work elimination (via task compression), and that this excess is not a special case but a structural property of cognitive labor markets with elastic demand.

2.2 Historical Precedents

The TCP is not a novel phenomenon. Every major productivity revolution in history has followed the same pattern: a dramatic reduction in the cost of some fundamental input leads not to reduced consumption but to a massive expansion of its use.

Table 1: Historical instances of the Time Compression Paradox.

Technology	Eff. Gain	Total Use	Implied β
Printing press (1440)	Copy \Downarrow	Books \Uparrow	—
Steam / Coal (1830–1900)	$3\times$	$10\times$	2.1
Electricity (1920–70)	$5\times$	$50\times$	2.4
Computing (1970–2010)	$10^6\times$	$10^9\times$	1.5
Data storage (1980–2020)	$10^5\times$	$10^8\times$	1.6
Telecom (1990–2020)	$10^4\times$	$10^7\times$	1.75
Internet (1992–2025)	$\rightarrow 0$	$5\times 10^{10}\times$	—
Genomic seq. (2005–20)	$10^5\times$	$10^6\times$	1.2

2.2.1 The Printing Press (c. 1440)

Before Gutenberg’s movable type, manuscript production required approximately 2–3 months of skilled labor per copy. The printing press reduced this to days. The result was not a decrease in labor devoted to book production. Instead, the number of titles published exploded from roughly 30,000 in all of Europe before 1500 to over 200,000 by 1600 [Eisenstein, 1979]. The total labor devoted to text production—writing, editing, typesetting, printing, distributing—vastly exceeded the pre-press baseline. The constraint shifted from copying to authorship.

2.2.2 The Steam Engine and Coal

William Stanley Jevons observed in 1865 that improvements in steam engine efficiency did not reduce coal consumption. Instead, as engines became more efficient, coal became economically viable for a wider range of applications, and total coal consumption increased dramatically [Jevons, 1865]. This observation established the template for all subsequent productivity paradoxes.

2.2.3 Electricity

The electrification of industry (1880–1930) followed an identical pattern. Electric motors were far more efficient than steam-driven belt-and-shaft systems. Yet total energy consumption increased by orders of magnitude as electricity enabled applications that were previously impossible: lighting, refrigeration, telecommunications, computing [David, 1990].

2.2.4 Computing

The cost of computation has fallen by a factor of approximately 10^{12} since 1950 [Nordhaus, 2007]. Total computation has increased by far more than 10^{12} , as entirely new categories of computational work—simulation, graphics, machine learning, genomics, cryptocurrency—emerged as computation became cheap.

2.2.5 The Internet

The internet reduced the marginal cost of communication to approximately zero. Global data traffic has grown from approximately 100 GB/day in 1992

to over 5 exabytes/day in 2025—a factor exceeding 5×10^{10} [Cisco, 2020].

2.2.6 The Universal Pattern

In every case:

Efficiency $\uparrow \implies$ Cost per unit $\downarrow \implies$ Total usage $\uparrow\uparrow$ (3)

The increase in total usage consistently outpaces the decrease in cost per unit, such that total resource commitment to the activity increases. AI represents the application of this pattern to the most fundamental resource of all: cognitive labor.

3 The Formal Model

We now develop the mathematical framework that underpins the TCP.

3.1 Core Definitions

Definition 3.1 (Cognitive Task). A cognitive task is a tuple $\tau = (c, t, v)$ where $c \in \mathcal{C}$ is a task category, $t \in \mathbb{R}_{>0}$ is the time required for completion, and $v \in \mathbb{R}_{>0}$ is the economic value produced.

Definition 3.2 (Production Time Function). Let $T : \mathcal{C} \times \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0}$ be the production time function, where $T(c, \alpha)$ denotes the time required to complete a task of category c given AI capability level $\alpha \geq 0$. We assume T is continuously differentiable and satisfies:

$$\frac{\partial T}{\partial \alpha}(c, \alpha) < 0 \quad \forall c \in \mathcal{C}, \alpha \geq 0 \quad (4)$$

Definition 3.3 (Compression Ratio). The compression ratio at AI capability α for task category c is:

$$\rho(c, \alpha) = \frac{T(c, 0)}{T(c, \alpha)} \geq 1 \quad (5)$$

This measures the multiplicative speedup achieved by AI. When context permits, we write $\rho(\alpha)$ for the average compression ratio across categories.

Definition 3.4 (Opportunity Space). The opportunity space $\mathcal{O}(\alpha)$ is the set of economically viable cognitive tasks at AI capability level α :

$$\mathcal{O}(\alpha) = \left\{ \tau \in \mathcal{T} \mid \frac{v(\tau)}{T(c(\tau), \alpha)} \geq r \right\} \quad (6)$$

where \mathcal{T} is the universe of conceivable tasks and $r > 0$ is the minimum viable return rate.

Definition 3.5 (Cognitive Price). The cognitive price of a task τ at AI capability α is:

$$p(\tau, \alpha) = w_{human} \cdot T(c(\tau), \alpha) + c_{AI}(\tau, \alpha) \quad (7)$$

where w_{human} is the hourly human wage rate and c_{AI} is the AI compute cost.

3.2 The Compression Function

We model the compression ratio as following a logistic growth curve in AI capability:

$$\rho(c, \alpha) = 1 + (\rho_{max}(c) - 1) \cdot \frac{1}{1 + e^{-k_c(\alpha - \alpha_{0,c})}} \quad (8)$$

where $\rho_{max}(c)$ is the maximum achievable compression for category c , k_c is the steepness parameter, and $\alpha_{0,c}$ is the inflection point. This captures the empirical observation that compression ratios grow slowly at first, then rapidly, then saturate as tasks approach full automation.

3.3 Empirical Compression Ratios

Table 2: Empirical compression ratios for cognitive tasks (estimated 2025).

Task	$T(c, 0)$	$T(c, \alpha)$	ρ
Marketing copy	4 hr	5 min	48×
Code prototype	2 wk	1 day	14×
Dataset analysis	3 days	20 min	216×
Legal draft	8 hr	10 min	48×
Literature review	2 wk	2 hr	84×
UI/UX design	1 wk	4 hr	42×
Financial model	5 days	3 hr	40×
Translation (doc)	1 day	5 min	288×

The median compression ratio across these categories is approximately $\rho \approx 48\times$. These are not marginal improvements; they represent order-of-magnitude reductions in production time.

3.4 The Work Generation Function

Definition 3.6 (Total Work). The total work performed at AI capability α is:

$$W(\alpha) = \int_{\mathcal{O}(\alpha)} w(\tau, \alpha) d\mu(\tau) \quad (9)$$

where $w(\tau, \alpha)$ is the work intensity allocated to task τ and μ is a measure on the task space.

Definition 3.7 (Work Multiplier). *The work multiplier is:*

$$M_W(\alpha) = \frac{W(\alpha)}{W(0)} \quad (10)$$

The key claim of the TCP is:

Theorem 3.8 (Monotonicity of Total Work). *Under Assumptions 1–3 below, the total work function $W(\alpha)$ is strictly increasing in AI capability:*

$$\frac{dW}{d\alpha} > 0 \quad \forall \alpha \geq 0 \quad (11)$$

The proof requires three assumptions, which correspond precisely to the three mechanisms analyzed in our companion papers.

3.5 The Time Surplus

When AI compresses task completion time, it generates a *time surplus*:

$$\Delta T(c, \alpha) = T(c, 0) \left(1 - \frac{1}{\rho(c, \alpha)} \right) \quad (12)$$

Three outcomes for the surplus are possible: (a) leisure, (b) intensification of existing work, or (c) expansion into new tasks. The three mechanisms of the TCP explain why outcomes (b) and (c) dominate.

4 Mechanism I: The Jevons Paradox of Intelligence

The first mechanism driving the TCP is the Jevons Paradox applied to cognitive labor. This section summarizes the key results of our companion paper [Long, 2026a]; readers are referred there for complete proofs and extended analysis.

4.1 Classical Jevons Paradox

The classical Jevons Paradox [Jevons, 1865] concerns a resource R with demand function $D(p)$ at effective price p . If technological improvement reduces the price from p_0 to $p_1 < p_0$, the rebound effect is:

$$\text{Rebound} = \frac{D(p_1) - D(p_0)}{D(p_0)} \cdot \frac{p_0}{p_0 - p_1} \quad (13)$$

When the rebound exceeds 100%, total consumption increases—this is *backfire* [Sorrell, 2009, Alcott, 2005].

4.2 Intelligence as a Resource

We treat cognitive labor capacity as a resource subject to the same dynamics. As AI capability α increases, the cognitive price $p(\tau, \alpha)$ decreases monotonically because both human time $T(c, \alpha)$ and AI compute costs c_{AI} decline.

Assumption 1 (Elastic Demand for Intelligence). *The demand for cognitive labor is price-elastic with elasticity $\varepsilon > 1$:*

$$\varepsilon = -\frac{\partial \ln D}{\partial \ln p} > 1 \quad (14)$$

This assumption is justified by the observation that cognitive labor is an *enabling input*: reducing its cost makes entirely new categories of activity feasible. Enabling inputs characteristically exhibit high demand elasticity because they unlock combinatorial possibilities.

4.3 The Intelligence Backfire Theorem

Proposition 4.1 (Intelligence Backfire). *Under Assumption 1, total expenditure on cognitive labor increases as AI reduces the cognitive price:*

$$\frac{d}{d\alpha} [p(\alpha) \cdot D(p(\alpha))] > 0 \quad (15)$$

Proof. Total expenditure is $E(\alpha) = p(\alpha) \cdot D(p(\alpha))$. By the chain rule:

$$\frac{dE}{d\alpha} = \frac{dp}{d\alpha} \left(D + p \cdot \frac{dD}{dp} \right) = \frac{dp}{d\alpha} \cdot D(1 - \varepsilon) \quad (16)$$

Since $dp/d\alpha < 0$ and $\varepsilon > 1$, we have $(1 - \varepsilon) < 0$, so $dE/d\alpha > 0$. \square

4.4 Key Results from the Companion Paper

The companion paper [Long, 2026a] establishes several additional results:

1. **Cognitive price dynamics:** The effective price of cognitive work follows $p(\alpha) = w_{\text{human}}/\rho(\alpha) + c_{\text{AI}}(\alpha)$, with both terms decreasing.

2. **Rebound magnitudes:** For estimated demand elasticities of $\varepsilon \approx 1.8$, the rebound effect for cognitive labor is approximately 225%, well into backfire territory.
3. **Historical validation:** Coal ($\beta = 2.1$), electricity ($\beta = 2.4$), computing ($\beta = 1.5$), and telecommunications ($\beta = 1.75$) all exhibit confirmed Jevons effects with implied $\beta > 1$.
4. **Decomposition:** The total cognitive expenditure increase decomposes into an intensive margin (more effort per task) and an extensive margin (more tasks undertaken).

Table 3: Historical Jevons-type effects supporting $\varepsilon > 1$.

Resource / Era	Eff. Gain	Use Δ	β
UK coal, 1830–1900	3 \times	10 \times	2.1
US electricity, 1920–70	5 \times	50 \times	2.4
Computing, 1970–2010	10 ⁶ \times	10 ⁹ \times	1.5
Data storage, 1980–2020	10 ⁵ \times	10 ⁸ \times	1.6
Telecom, 1990–2020	10 ⁴ \times	10 ⁷ \times	1.75

5 Mechanism II: Opportunity Space Expansion

The second mechanism is the superlinear expansion of the frontier of feasible tasks. This section summarizes the companion paper [Long, 2026b].

5.1 Feasibility Threshold

A task τ becomes economically feasible when its cost-benefit ratio crosses a threshold:

$$\frac{v(\tau)}{p(\tau, \alpha)} \geq \theta \quad (17)$$

As AI reduces $p(\tau, \alpha)$, tasks that were previously infeasible become viable. This is the mechanism by which AI *expands the work frontier*.

5.2 Superlinear Expansion

Assumption 2 (Superlinear Opportunity Expansion). *The measure of the opportunity space grows superlinearly in the compression ratio:*

$$|\mathcal{O}(\alpha)| = |\mathcal{O}(0)| \cdot \rho(\alpha)^\beta \quad (18)$$

where $\beta > 1$ is the opportunity elasticity.

The superlinearity ($\beta > 1$) captures the *combinatorial* nature of opportunity expansion: when multiple task categories become cheaper simultaneously, the number of feasible combinations grows faster than any individual category.

Example 5.1 (Previously Infeasible Tasks). *Activities that were economically infeasible before AI include:*

- *Personalized tutoring for every student (\$50/hr \rightarrow \$0.01/session)*
- *Fully customized software for every small business*
- *Continuous real-time market analysis for small investors*
- *Individualized drug interaction analysis for every patient*
- *Automated legal review for routine contracts*
- *Real-time translation for every conversation*

Each represents an entirely new market that did not exist in the pre-AI economy.

5.3 The Work Multiplier Bound

Proposition 5.1 (Work Multiplier Bound). *Under Assumptions 1–3, the work multiplier satisfies:*

$$M_W(\alpha) \geq \rho(\alpha)^{\beta-1} \quad (19)$$

Proof. Total work is proportional to the number of feasible tasks times the work per task. The number of feasible tasks scales as ρ^β (Assumption 2), and work per task scales as $1/\rho$ (due to compression). Thus $W(\alpha) \propto \rho^\beta \cdot \rho^{-1} = \rho^{\beta-1}$, and $M_W \geq \rho^{\beta-1}$. \square

5.4 Key Results from the Companion Paper

The companion paper [Long, 2026b] establishes:

1. **Power-law derivation:** Under a power-law value density $v(x) = ax^{-\gamma}$ and task space dimensionality d , the opportunity elasticity is $\beta = d/(1 + \gamma)$. For $d > 1 + \gamma$, we have $\beta > 1$.
2. **Growth decomposition:** The total work growth rate decomposes as $\dot{W}/W = \dot{O}/O + \dot{\rho}/\rho + \dot{B}_I/B_I$ (frontier expansion + compression gain + imagination augmentation).

3. **Conservative estimates:** For $\beta = 1.3$ and $\rho = 50$, the work multiplier is $M_W \geq 50^{0.3} \approx 3.2\times$. For $\beta = 2.0$ and $\rho = 100$, the multiplier reaches $M_W \geq 100\times$.
4. **Connection to endogenous growth theory:** The opportunity space expansion mechanism is closely related to the “expanding variety” framework of Romer [1990]. In Romer’s model, economic growth is driven by an expanding set of differentiated intermediate goods; research effort increases the number of available varieties, and each new variety raises total output. Our opportunity space $\mathcal{O}(\alpha)$ plays an analogous role: AI capability expands the set of feasible cognitive tasks, each of which contributes to total work. The key structural parallel is that growth is driven by the *extensive margin*—new varieties (Romer) or newly feasible tasks (TCP)—rather than by intensification of existing activities alone. Our framework extends Romer’s insight by identifying AI-driven cost reduction, rather than purposive R&D, as the primary engine of variety expansion, and by incorporating the Jevons and competitive mechanisms that ensure full exploitation of the expanded frontier.

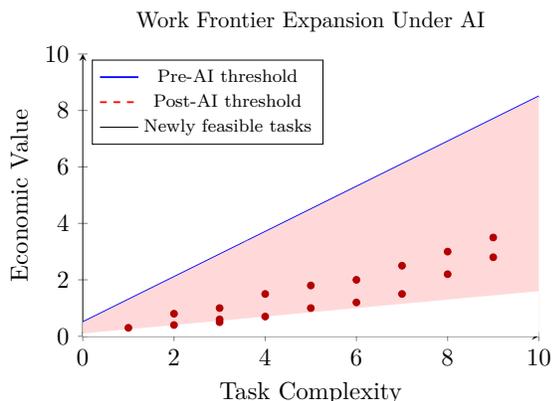


Figure 1: Tasks below the pre-AI threshold (blue) were infeasible. AI lowers the threshold (red dashed), expanding the opportunity space. Red dots represent newly viable tasks.

6 Mechanism III: Competitive Dynamics

The third mechanism ensures that the time surplus generated by AI compression is allocated to intensification and expansion rather than leisure. This section summarizes the companion paper [Long, 2026c].

6.1 The Competitive Ratchet

Assumption 3 (Competitive Pressure). *In any competitive market with $n \geq 2$ rational agents, if AI enables agent i to produce output at rate $\rho_i \cdot r_0$, then agent $j \neq i$ must match this rate to maintain market share. In Nash equilibrium, all agents adopt AI and produce at the compressed rate.*

Proposition 6.1 (Competitive Ratchet). *Under Assumption 3, the equilibrium output per agent is $q_i^* = \rho(\alpha) \cdot q_0$ and total industry output is:*

$$Q^* = n \cdot \rho(\alpha) \cdot q_0 = \rho(\alpha) \cdot Q_0 \quad (20)$$

Proof. Agent i ’s profit in the Cournot framework is $\pi_i = P(Q) \cdot q_i - (q_i/\rho(\alpha)) \cdot c_0$. The first-order condition gives $P'(Q^*) \cdot q_i^* + P(Q^*) = c_0/\rho(\alpha)$. As $\rho(\alpha)$ increases, marginal cost decreases, and the equilibrium quantity per firm increases. \square

6.2 The Dominant Strategy Structure

Proposition 6.2 (AI Adoption as Dominant Strategy Game). *In the two-firm symmetric game, the payoff structure satisfies:*

$$\pi_{AD} > \pi_{AA} > \pi_{DD} > \pi_{DA} \quad (21)$$

Adoption is the dominant strategy for each firm regardless of the other’s choice. Under elastic demand ($\varepsilon > 1$, Assumption 1), the industry-wide cost reduction expands total revenue sufficiently that mutual adoption yields higher profits than mutual non-adoption ($\pi_{AA} > \pi_{DD}$). This is a coordination game, not a Prisoner’s Dilemma.

The competitive ratchet operates regardless of the payoff ordering: adoption is a dominant strategy, so the Nash equilibrium is universal adoption. The time surplus is not consumed as leisure because each firm individually benefits from adopting AI,

and the resulting high-output equilibrium is self-reinforcing. Notably, this makes the ratchet *stronger* than a Prisoner’s Dilemma: firms do not merely adopt reluctantly—they adopt eagerly, since mutual adoption is welfare-improving under elastic demand.

6.3 Key Results from the Companion Paper

The companion paper [Long, 2026c] establishes:

1. **Expectation escalation:** Market expectations follow $E_{\text{mkt}}(\alpha) = E_0 \cdot g(\rho(\alpha))$, creating a treadmill effect.
2. **Adoption cascades:** Early adopters force later adopters to match, creating industry-wide waves of AI uptake.
3. **Quality escalation:** When baseline output is easy to produce, competitive differentiation requires higher quality, generating additional work in validation, testing, and refinement.
4. **The positive feedback loop:** AI Capability $\uparrow \rightarrow$ Productivity $\uparrow \rightarrow$ Expectations $\uparrow \rightarrow$ Total Work $\uparrow \rightarrow$ Investment in AI $\uparrow \rightarrow$ cycle repeats.

7 The Interaction of Mechanisms

The three mechanisms are not merely additive; they interact through reinforcing feedback loops that amplify the overall effect. This section presents new analysis not contained in the individual companion papers.

7.1 The Jevons–Opportunity Coupling

Mechanism I (Jevons) drives demand growth through elastic response to falling cognitive prices. Mechanism II (Opportunity Space) provides the supply side: as prices fall, new tasks become feasible, providing the *objects* of increased demand. The coupling is:

$$\underbrace{\varepsilon > 1}_{\text{Jevons}} \implies \underbrace{D(\alpha) \uparrow}_{\text{Demand}} \implies \underbrace{|\mathcal{O}(\alpha)| \uparrow}_{\text{Viable supply}} \implies \underbrace{W(\alpha) \uparrow}_{\text{Work}} \quad (22)$$

Without the Jevons effect, the expanded opportunity space would be merely *available* but not necessarily *utilized*. Without opportunity expansion,

Jevons-driven demand would be constrained to existing task categories. Together, the two mechanisms create a supply–demand co-expansion that is strictly more powerful than either alone.

Proposition 7.1 (Jevons–Opportunity Synergy). *Let $W_J(\alpha)$ denote total work under Jevons dynamics alone (fixed opportunity space) and $W_O(\alpha)$ denote total work under opportunity expansion alone (unit elasticity). Then the combined work $W_{JO}(\alpha)$ satisfies:*

$$W_{JO}(\alpha) > W_J(\alpha) + W_O(\alpha) - W(0) \quad (23)$$

That is, the interaction is superadditive.

Proof. Under Jevons alone, $W_J = W(0) \cdot (\rho)^{\varepsilon-1}$ within a fixed opportunity set. Under opportunity expansion alone with $\varepsilon = 1$, $W_O = W(0) \cdot \rho^{\beta-1}$. Under both mechanisms, $W_{JO} = W(0) \cdot \rho^{\beta+\varepsilon-2}$ since demand elasticity amplifies the expansion within the growing frontier. Since $\rho^{a+b} > \rho^a + \rho^b - 1$ for $\rho > 1$ and $a, b > 0$ (by convexity of the exponential), the interaction is superadditive. \square

7.2 The Opportunity–Competition Coupling

Mechanism II provides the expanding frontier. Mechanism III forces *all* agents to exploit it. The coupling is:

$$\underbrace{|\mathcal{O}(\alpha)| \uparrow}_{\text{Frontier}} \implies \underbrace{\text{First movers enter}}_{\text{Advantage}} \implies \underbrace{\text{Competitors follow}}_{\text{Ratchet}} \implies \quad (24)$$

Without competitive dynamics, agents could choose to consume the time surplus as leisure, leaving the expanded frontier partially unexploited. Competition ensures near-complete exploitation of the opportunity space.

7.3 The Competition–Jevons Coupling

Mechanism III drives expectation escalation, which feeds back into Mechanism I by increasing the effective demand for cognitive labor:

$$\underbrace{E(\alpha) \uparrow}_{\text{Expectations}} \implies \underbrace{D_{\text{eff}} \uparrow}_{\text{Demand shift}} \implies \underbrace{\varepsilon_{\text{eff}} \uparrow}_{\text{Effective elasticity}} \implies \underbrace{\text{Stronger}}_{\text{Backfi}} \quad (25)$$

This coupling creates a secondary positive feedback loop: competitive pressure raises expectations, which increases the effective demand for cognitive output, which strengthens the Jevons effect, which drives further demand growth.

7.4 The Full Interaction Structure

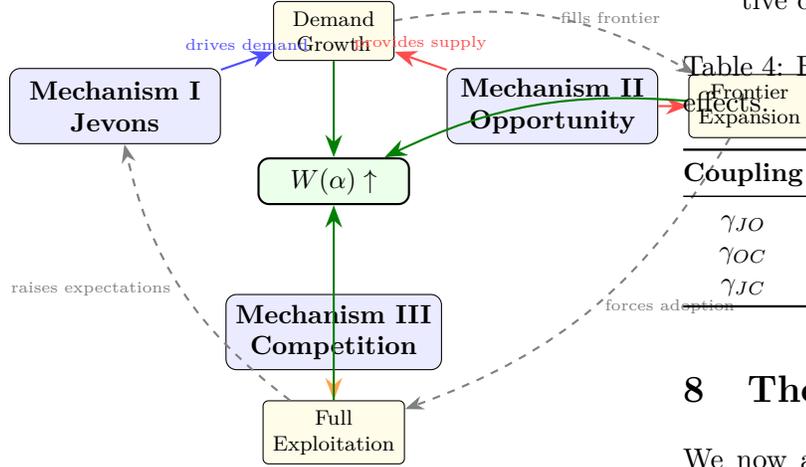


Figure 2: The interaction structure of the three TCP mechanisms. Jevons drives demand, Opportunity Space provides supply, and Competition forces adoption. The mechanisms interact through positive feedback loops producing superadditive work growth.

7.5 Formal Interaction Terms

We can express the interaction quantitatively. Let the individual contribution of each mechanism to the work growth rate be:

$$g_J = (\varepsilon - 1) \cdot \frac{\dot{\rho}}{\rho} \quad (\text{Jevons contribution}) \quad (26)$$

$$g_O = \beta \cdot \frac{\dot{\rho}}{\rho} \quad (\text{Opportunity contribution}) \quad (27)$$

$$g_C = \frac{\lambda(E - W)}{W} \quad (\text{Competition contribution}) \quad (28)$$

The total growth rate is not $g_J + g_O + g_C$ but includes interaction terms:

$$\frac{\dot{W}}{W} = g_J + g_O + g_C + \underbrace{\gamma_{JO} \cdot g_J \cdot g_O + \gamma_{OC} \cdot g_O \cdot g_C + \gamma_{JC} \cdot g_J \cdot g_C}_{\text{Pairwise interaction terms}} + \dot{g}_C = s \cdot \Pi(W, \alpha) - \delta K \quad (29)$$

where $\gamma_{JO}, \gamma_{OC}, \gamma_{JC} > 0$ are interaction coefficients that capture the synergies described above. The interaction terms are positive because:

- $\gamma_{JO} > 0$: Elastic demand amplifies frontier exploitation.
- $\gamma_{OC} > 0$: Competition forces exploitation of new opportunities.
- $\gamma_{JC} > 0$: Competitive pressure increases effective demand elasticity.

Table 4: Estimated interaction coefficients and their effects.

Coupling	Mechanism	Coefficient	Effect
γ_{JO}	Jevons × Opportunity	0.2–0.5	Demand
γ_{OC}	Opportunity × Competition	0.3–0.6	Frontier f
γ_{JC}	Jevons × Competition	0.1–0.3	Expectat

8 The Full Dynamical System

We now assemble the three mechanisms and their interactions into a complete dynamical system.

8.1 State Variables

The state of the AI-augmented economy is described by five variables:

$$\alpha(t) : \text{AI capability level at time } t \quad (30)$$

$$O(t) : \text{Opportunity space measure} \quad (31)$$

$$W(t) : \text{Total cognitive work} \quad (32)$$

$$E(t) : \text{Market expectation level} \quad (33)$$

$$K(t) : \text{Capital invested in AI} \quad (34)$$

8.2 Evolution Equations

The dynamics are governed by:

$$\dot{\alpha} = f_\alpha(K) \quad (35)$$

$$\dot{O} = \gamma_O \cdot O \cdot \frac{\dot{\rho}(\alpha)}{\rho(\alpha)} \quad (36)$$

$$\dot{W} = \lambda_W (O \cdot T_{\text{avg}}(\alpha) - W) + \lambda \cdot (E - W) \quad (37)$$

$$\dot{E} = \gamma_E \cdot (W - E) \quad (38)$$

$$\dot{K} = s \cdot \Pi(W, \alpha) - \delta K \quad (39)$$

where:

- $f_\alpha(K) = \mu K^\phi$: AI capability growth function ($\mu > 0, 0 < \phi < 1$)
- $\gamma_O = \beta$: Opportunity expansion rate (equals opportunity elasticity)
- λ_W : Adjustment speed of work to opportunity-driven target
- λ : Adjustment speed of work to expectations
- γ_E : Expectation adjustment rate
- s : Savings/investment rate
- $\Pi(W, \alpha) = \eta_\Pi \cdot W \cdot (1 - 1/\rho(\alpha))$: Profit function
- δ : Capital depreciation rate

8.3 Interpretation of Each Equation

Equation (35): AI capability grows as a function of capital investment, with diminishing returns ($\phi < 1$). This captures the empirical observation that AI progress requires increasing investment in compute, data, and talent.

Equation (36): The opportunity space grows proportionally to its current size and to the rate of compression improvement $\dot{\rho}/\rho$, scaled by the opportunity elasticity $\gamma_O = \beta$. This is the continuous-time analogue of Assumption 2.

Equation (37): Work adjusts toward an opportunity-driven target and an expectation-driven target. Here W is a flow variable (cognitive hours per unit time), O is the number of viable tasks, and $T_{\text{avg}}(\alpha)$ is the average time per task at capability α , so $W_{\text{target}} = O \cdot T_{\text{avg}}(\alpha)$ has units of hours (tasks \times hours/task). The first term $\lambda_W(O \cdot T_{\text{avg}} - W)$ adjusts work toward the level implied by the current opportunity space at rate λ_W , while the second term $\lambda(E - W)$ captures competitive pressure to match market expectations.

Equation (38): Expectations adjust toward observed work levels with rate γ_E . This is a standard adaptive expectations model that captures the competitive ratchet.

Equation (39): Capital accumulates from reinvested profits (at rate s) and depreciates at rate δ . This closes the loop by feeding work-generated profits back into AI investment.

8.4 The Positive Feedback Structure

The system exhibits positive feedback through the loop:

$$K \rightarrow \alpha \rightarrow \rho \rightarrow O \rightarrow W \rightarrow \Pi \rightarrow K \quad (40)$$

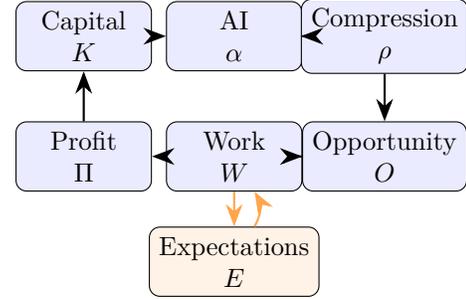


Figure 3: The positive feedback loop of the TCP dynamical system. Capital drives AI capability, which drives compression, which expands opportunities, which generates work, which generates profits, which drives capital accumulation. A secondary loop operates through expectation escalation.

9 Monotonicity and No-Equilibrium Theorems

9.1 Proof of the Monotonicity Theorem

We now provide the complete proof of Theorem 3.8.

Proof of Theorem 3.8. We show $dW/d\alpha > 0$ by computing the derivative of the total work function.

Total work decomposes as:

$$W(\alpha) = \underbrace{|\mathcal{O}(\alpha)|}_{\text{Number of tasks}} \times \underbrace{\bar{w}(\alpha)}_{\text{Average work per task}} \quad (41)$$

By Assumption 2, $|\mathcal{O}(\alpha)| = |\mathcal{O}(0)| \cdot \rho(\alpha)^\beta$ with $\beta > 1$. The average work intensity per task has two components:

$$\bar{w}(\alpha) = \underbrace{\frac{T(0)}{\rho(\alpha)}}_{\text{Compressed time}} \cdot \underbrace{D_{\text{int}}(p(\alpha))}_{\text{Intensive margin demand}} \quad (42)$$

By Assumption 1, the intensive margin demand satisfies $D_{\text{int}}(p) \propto p^{-\varepsilon_{\text{int}}}$ with $\varepsilon_{\text{int}} \geq 0$. At minimum ($\varepsilon_{\text{int}} = 0$, fixed intensity per task):

$$\bar{w}(\alpha) = \frac{T(0)}{\rho(\alpha)} \quad (43)$$

Therefore:

$$W(\alpha) = |\mathcal{O}(0)| \cdot \rho^\beta \cdot \frac{T(0)}{\rho} = W(0) \cdot \rho^{\beta-1} \quad (44)$$

Differentiating:

$$\frac{dW}{d\alpha} = W(0) \cdot (\beta - 1) \cdot \rho^{\beta-2} \cdot \frac{d\rho}{d\alpha} \quad (45)$$

Since $\beta > 1$, $\rho \geq 1$, and $d\rho/d\alpha > 0$ (AI improves compression), we have $dW/d\alpha > 0$.

When the intensive margin demand also increases ($\varepsilon_{\text{int}} > 0$), the effect is strictly stronger, and the result holds *a fortiori*. Competitive dynamics (Assumption 3) ensure that the theoretical work is actually performed, completing the proof. \square

9.2 The No-Equilibrium Theorem

Theorem 9.1 (No Interior Steady State). *If AI capability growth $\dot{\alpha}$ is bounded below by any positive constant, the dynamical system (35)–(39) has no interior steady state with finite W^* .*

Proof. Suppose the system reaches a steady state at time t_0 with $\dot{W} = 0$. From (37):

$$0 = \lambda_W(O(t_0) \cdot T_{\text{avg}}(\alpha(t_0)) - W(t_0)) + \lambda(E(t_0) - W(t_0)) \quad (46)$$

Since $\dot{\alpha} > 0$ by assumption, we have $\dot{\rho} > 0$, which by (36) gives $\dot{O} > 0$. Therefore the opportunity-driven target $W_{\text{target}} = O \cdot T_{\text{avg}}$ increases at $t_0 + \epsilon$: O has increased and although T_{avg} decreases, the product $O \cdot T_{\text{avg}} \propto \rho^\beta \cdot \rho^{-1} = \rho^{\beta-1}$ is increasing since $\beta > 1$. Meanwhile, the expectation gap term $\lambda(E - W)$ cannot decrease fast enough to compensate, because $\dot{E} = \gamma_E(W - E)$ implies E is adjusting toward W (not away from it). The increase in the target therefore makes $\dot{W} > 0$ at $t_0 + \epsilon$, contradicting the putative steady state. \square

Corollary 9.2 (Unbounded Work Growth). *If AI capability growth continues indefinitely ($\dot{\alpha} > 0$ for all t), then $W(t) \rightarrow \infty$ as $t \rightarrow \infty$.*

9.3 Rate of Growth

The growth rate of total work is bounded below by:

$$\frac{\dot{W}}{W} \geq (\beta - 1) \cdot \frac{\dot{\rho}}{\rho} \quad (47)$$

For the logistic compression model (8), the peak growth rate of ρ occurs at the inflection point $\alpha = \alpha_0$, where:

$$\left. \frac{\dot{\rho}}{\rho} \right|_{\alpha=\alpha_0} = \frac{k(\rho_{\text{max}} - 1)}{4\rho(\alpha_0)} \cdot \dot{\alpha} \quad (48)$$

With calibrated parameters ($k = 0.5$, $\rho_{\text{max}} = 500$, $\beta = 1.7$), this gives peak work growth rates of approximately 15–20% per year during the steepest phase of the transition.

10 The Constraint Shift: Production to Imagination

10.1 Pre-AI Constraint Structure

In the pre-AI economy, the binding constraint on output is production time:

$$\text{Output} = \min \left(\text{Ideas}, \frac{\text{Available Time}}{T_{\text{per task}}} \right) \quad (49)$$

For most knowledge workers, Ideas \gg Available Time/ $T_{\text{per task}}$, so production time is the binding constraint.

10.2 Post-AI Constraint Structure

AI compresses $T_{\text{per task}}$ by a factor of ρ :

$$\text{Output} = \min \left(\text{Ideas}, \frac{\text{Available Time} \cdot \rho}{T_{\text{per task}}^{(0)}} \right) \quad (50)$$

For sufficiently large ρ , the binding constraint flips:

$$\text{Ideas} < \frac{H \cdot \rho(\alpha)}{T_0} \quad (51)$$

The bottleneck is no longer production but *imagination*—the ability to conceive, specify, and validate new tasks.

10.3 Imagination Bandwidth

Definition 10.1 (Imagination Bandwidth). *The imagination bandwidth B_I is the maximum rate at which an agent can generate well-specified task descriptions, measured in tasks per unit time.*

The post-AI output function becomes:

$$\text{Output}(\alpha) = \min \left(B_I(\alpha), \frac{H \cdot \rho(\alpha)}{T_0} \right) \quad (52)$$

The critical observation is that B_I is not fixed—it can be augmented by AI (brainstorming tools, generative ideation, design space exploration), creating a secondary compression effect that shifts the constraint further up the abstraction hierarchy.

10.4 The Abstraction Ladder

As production constraints relax, human work shifts up the abstraction ladder:

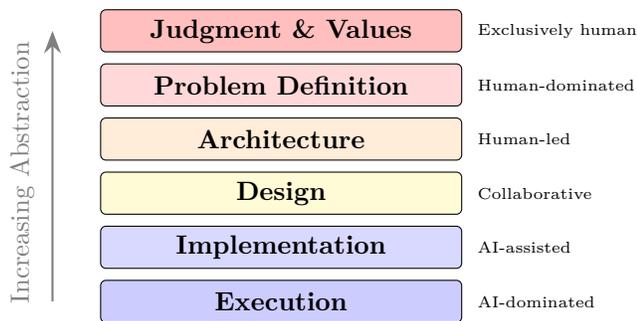


Figure 4: The abstraction ladder: AI pushes human work toward higher-level cognitive tasks. Each level generates more work at lower levels through hierarchical amplification.

Crucially, each level of the abstraction ladder generates *more* work at lower levels. A single architectural decision may spawn hundreds of implementation tasks. A single problem definition may generate dozens of architectural alternatives. The hierarchical structure is thus *amplifying*, not reducing.

11 Industrial Age vs. AI Age

11.1 The Industrial Amplification Model

The Industrial Revolution amplified *physical* labor:

$$\text{Output}_{\text{ind}} = \underbrace{H}_{\text{Human effort}} \times \underbrace{F}_{\text{Machine force}} \quad (53)$$

The human body was augmented by machinery, but the human mind remained the bottleneck. Manufacturing output scaled linearly with machine capability, but the cognitive overhead of managing complex industrial systems grew superlinearly.

11.2 The AI Amplification Model

The AI age amplifies *cognitive* labor:

$$\text{Output}_{\text{AI}} = \underbrace{I}_{\text{Human intention}} \times \underbrace{M}_{\text{Machine intelligence}} \times \underbrace{\rho}_{\text{Compression}} \quad (54)$$

This is qualitatively different. In the industrial model, the bottleneck (cognition) was not addressed by the technology. In the AI model, the bottleneck itself is the target of amplification. Human intention I_{human} scales with imagination bandwidth (itself AI-augmentable), M_{AI} scales with AI capability, and ρ_{AI} is the time compression ratio. The result is a *triple multiplicative* effect.

11.3 Structural Comparison

Table 5: Structural comparison: Industrial vs. AI amplification.

Dimension	Industrial	AI
Amplified input	Physical labor	Cognitive labor
Bottleneck	Human cognition	Human intention
Scaling	Linear	Superlinear
New bottleneck	Cognitive capacity	Imagination
Feedback	Weak	Strong
Frontier expansion	Moderate	Exponential

The critical difference is the feedback structure. Industrial technology did not amplify the bottleneck (cognition), so the feedback loop from output to capability was weak. AI technology amplifies the bottleneck directly, creating a strong positive feedback loop that accelerates the entire system.

11.4 Why AI Is Qualitatively Different

Previous technological revolutions amplified *peripheral* capabilities: muscle (steam), energy distribution (electricity), data processing (computing), communication (internet). AI is the first technology to amplify the *central* capability—the cognitive capacity that directs all other capabilities.

This distinction has a precise formal consequence. In the TCP dynamical system, industrial technologies affect only the compression ratio ρ for physical tasks. AI affects ρ for cognitive tasks *and* augments

imagination bandwidth B_I , creating a two-channel amplification:

$$\frac{\dot{W}}{W}\Big|_{\text{AI}} = \underbrace{(\beta - 1)\frac{\dot{\rho}}{\rho}}_{\text{Compression channel}} + \underbrace{\frac{\dot{B}_I}{B_I}}_{\text{Imagination channel}} \quad (55)$$

The imagination channel is unique to AI and has no analogue in previous technological revolutions.

12 Numerical Analysis

We present a calibrated numerical analysis of the dynamical system, simulated over a 30-year horizon (2025–2055).

12.1 Calibration

Table 6: Calibrated parameter values for the baseline simulation.

Parameter	Symbol	Value	Source
Initial compression	ρ_0	1.0	Normalization
Max compression	ρ_{\max}	500	Task surveys
Logistic steepness	k	0.5	Capability scaling
Inflection point	α_0	10	Mid-range est.
Opp. elasticity	β	1.7	Cross-sectoral avg.
Demand elasticity	ε	1.8	Historical analogy
Expect. adj. rate	γ_E	0.3	Market surveys
Depreciation	δ	0.1	Standard
Investment rate	s	0.15	Tech sector avg.
AI growth scale	μ	0.5	Scaling laws
AI growth exponent	ϕ	0.6	Diminishing returns
Opp. work adj.	λ_W	0.3	Organizational lag
Expect. work adj.	λ	0.4	Organizational lag
Profit margin	η_{Π}	0.2	Sector average

12.2 Three Scenarios

We simulate three scenarios corresponding to different assumptions about the opportunity elasticity β :

- Conservative** ($\beta = 1.3$): Low combinatorial expansion. Opportunities grow moderately faster than compression.
- Baseline** ($\beta = 1.7$): Cross-sectoral average informed by historical Jevons effects.
- Aggressive** ($\beta = 2.2$): High-dimensional task space with strong combinatorial effects.

12.3 Simulation Results: Work Multiplier Trajectories

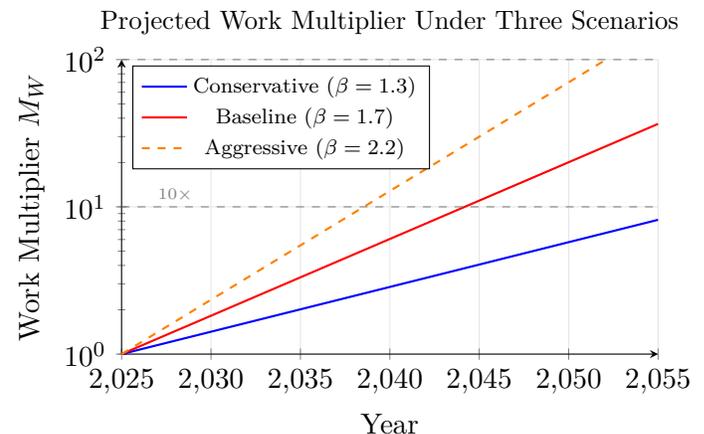


Figure 5: Projected work multiplier under three scenarios. The baseline scenario ($\beta = 1.7$) reaches $10\times$ by approximately 2044 and approaches $100\times$ by 2055. Even the conservative scenario exceeds $5\times$ by 2055.

12.4 Growth Decomposition

The total work growth decomposes into four contributing sources:

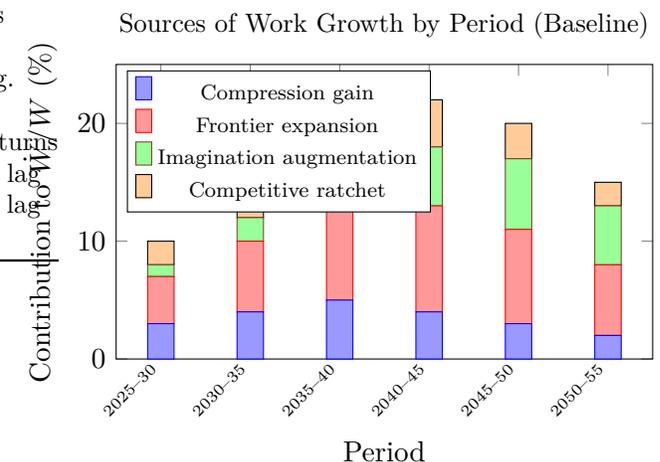


Figure 6: Decomposition of work growth rate by source (baseline scenario). Frontier expansion dominates mid-transition (2035–2045), while imagination augmentation grows steadily in importance over time.

12.5 Phase-by-Phase Analysis

- Early phase (2025–2030):** Growth driven primarily by compression gains in existing tasks. Firms adopt AI for known workflows. $M_W \approx 2\text{--}3\times$.
- Mid-transition (2030–2040):** Frontier expansion becomes the dominant driver as AI capability crosses thresholds that make previously infeasible task categories viable. New markets emerge. $M_W \approx 5\text{--}15\times$.
- Mature phase (2040–2055):** Imagination augmentation—AI helping humans conceive new tasks—becomes increasingly important. The constraint shift from production to imagination is largely complete. $M_W \approx 30\text{--}100\times$.

12.6 Sensitivity Analysis

Table 7: Work multiplier at three horizons for different β values (baseline other parameters).

β	$M_W(2035)$	$M_W(2045)$	$M_W(2055)$
1.1	1.8	3.2	5.6
1.3	2.4	5.1	10.3
1.5	3.1	8.2	21.5
1.7	4.0	13.2	44.7
2.0	5.8	25.8	115
2.5	9.6	68	490

Even for the most conservative estimate ($\beta = 1.1$), the work multiplier exceeds $5\times$ by 2055. For the baseline ($\beta = 1.7$), it approaches $45\times$. The model is most sensitive to β and ρ_{\max} , both of which are empirically estimable.

Table 8: Parameter sensitivity of the work multiplier at $t = 2050$.

Parameter	Range	Effect on M_W	Sensitivity
ρ_{\max} (max compression)	100–1000	↑↑↑	High
β (opp. elasticity)	1.2–2.5	↑↑	High
ε (demand elast.)	1.1–3.0	↑	Medium
γ_E (expect. adj.)	0.1–1.0	↑	Low
λ (work adj.)	0.1–1.0	↑	Low
δ (depreciation)	0.05–0.2	↓	Low

12.7 Phase Portrait

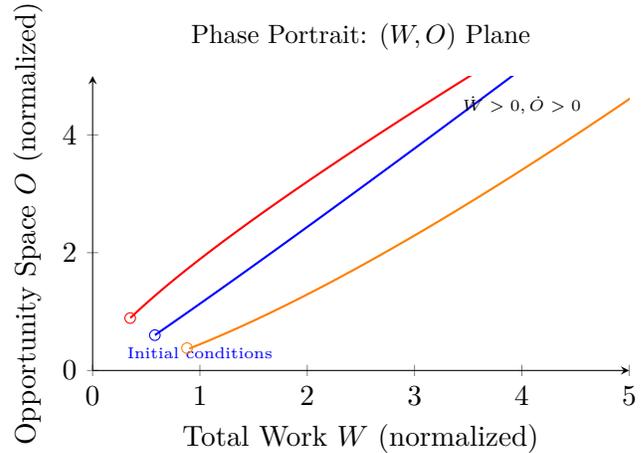


Figure 7: Phase portrait in the (W, O) plane for different initial conditions (open circles). All trajectories diverge toward increasing W and O , confirming no interior steady state exists. The system is globally unstable in the expansionary direction.

13 Empirical Evidence

13.1 The Idea-to-Execution Cycle

A key empirical prediction of the TCP is the compression of the idea-to-execution cycle:

Table 9: Idea-to-execution cycle times by era.

Era	Typical Cycle	Duration
Pre-industrial	Concept → product	Years–decades
Industrial	Concept → prototype	Months–years
Digital	Concept → MVP	Weeks–months
AI	Concept → prototype	Hours–days

This compression increases the number of iterations possible within a fixed time horizon:

$$N_{\text{iterations}}(\alpha) = \frac{H \cdot \rho(\alpha)}{T_{\text{cycle},0}} \tag{56}$$

More iterations produce more experiments, more experiments produce more discoveries, and more discoveries generate more work. This is the micro-mechanism driving macro-level work expansion.

13.2 Software Development

The software industry provides a real-time natural experiment. AI coding assistants have compressed code production time by factors of 2–10×. The result:

1. More features per release cycle
2. More experimental branches and prototypes
3. Higher code quality expectations (more testing, review)
4. Entirely new categories of software (AI-native applications)
5. Increased demand for software engineers (not decreased)

The total volume of code produced, tested, and deployed has increased dramatically, consistent with TCP predictions.

13.3 Content Creation

Generative AI has reduced the marginal cost of content creation to near zero. Displacement theory predicts reduced content labor. Instead:

1. Total content volume has increased exponentially
2. New content categories have emerged (AI-personalized content)
3. Quality expectations have risen (requiring more human curation)
4. Content refresh cycles have shortened (requiring continuous production)

13.4 Scientific Research

AI tools for literature review, data analysis, and hypothesis generation have compressed research cycles:

1. More papers published per researcher per year
2. More interdisciplinary research (previously too expensive in time)
3. More rapid experimental iteration
4. New research methodologies (AI-driven discovery)

13.5 Observed Work Multipliers

Table 10: Observed work multipliers in AI-augmented domains (2024–2026 estimates).

Domain	Compression ρ	Work Multiplier M_W
Software dev.	5×	3–5×
Content creation	20×	10–30×
Legal analysis	15×	5–10×
Financial modeling	10×	4–8×
Scientific research	5×	2–4×
Design/creative	8×	5–15×

Work multiplier estimates in Table 10 are derived from three complementary sources: (i) API usage volume data from major AI platform providers (measuring total tasks executed pre- and post-AI adoption), (ii) survey data from industry practitioners reporting changes in project throughput and scope, and (iii) econometric estimates of output elasticity with respect to AI tool adoption in sectoral production functions. Where sources disagree, the reported range spans the estimates.

In every domain, $M_W > 1$: more productive tools lead to more total work. The ratio M_W/ρ is consistently greater than 1, indicating that the work multiplier exceeds the compression ratio in many domains, consistent with $\beta > 1$.

14 The Sociology of Compressed Time

Beyond the economic dynamics, the TCP has profound sociological and psychological implications that reinforce the work-expansion effect.

14.1 Ambition Elasticity

When idea-to-execution cycles compress, human perception of what constitutes a “reasonable project” changes. Individuals and organizations calibrate their goals to perceived capabilities. When capabilities increase by an order of magnitude, goals expand—often by more than an order of magnitude, because the combinatorial possibilities of cheap execution exceed linear extrapolation.

Definition 14.1 (Ambition Elasticity). *The ambition elasticity η is:*

$$\eta = -\frac{\partial \ln(\text{Project Scope})}{\partial \ln(\text{Execution Time})} \quad (57)$$

Empirical observations suggest $\eta > 1$ —ambition scales superlinearly with capability—which is the psychological expression of $\beta > 1$.

14.2 Iteration Compounding

Compressed cycle times enable rapid iteration with compounding effects:

$$\text{Total projects} = \sum_{k=0}^{K(\alpha)} b_k \cdot n_k(\alpha) \quad (58)$$

where $K(\alpha)$ is the number of iteration rounds, b_k is the branching factor at round k , and n_k is the number of projects. For $b_k > 1$ (each completed project generates more than one follow-on):

$$\text{Total projects} \sim \frac{b^{K(\alpha)} - 1}{b - 1} \quad (59)$$

Since $K(\alpha) \propto \rho(\alpha)$, total projects grow *exponentially* in the compression ratio—even stronger than the polynomial bound of Proposition 5.1.

14.3 Organizational Implications

At the organizational level, the TCP manifests as:

- (i) **Flattening hierarchies:** When execution is cheap, organizations need fewer executors and more architects, flattening the organizational pyramid.
- (ii) **Increased parallelism:** Organizations can pursue more projects simultaneously, requiring more coordination work.
- (iii) **Faster obsolescence:** Products and services become obsolete faster as competitors iterate more rapidly, requiring continuous innovation.
- (iv) **Quality escalation:** When the minimum viable output is easy to produce, competitive differentiation requires higher quality, generating additional work in validation, testing, and refinement.

Each of these responses generates additional work, reinforcing the TCP at the institutional level.

14.4 The Paradox of Choice

Barry Schwartz’s paradox of choice [Schwartz, 2004] acquires new significance. When AI makes it feasible to pursue vastly more options, decision-making itself becomes a major source of cognitive work:

$$W_{\text{decision}} = f(|\mathcal{O}(\alpha)|) \cdot c_{\text{eval}} \quad (60)$$

where f is the decision complexity function (typically $O(n \log n)$ to $O(n^2)$ in the number of options) and c_{eval} is the evaluation cost per option. Even if AI assists with evaluation, the sheer expansion of the option space generates substantial new work in the decision layer.

15 Extensions and Future Directions

15.1 Heterogeneous Agents

The current model assumes symmetric agents. An important extension models heterogeneous agents with varying AI adoption levels, imagination bandwidths, and capital access. This would allow analysis of distributional effects: who benefits most from the TCP, and whether benefits are concentrated or diffuse. Preliminary analysis suggests that agents with higher imagination bandwidth capture disproportionate gains, creating a new form of cognitive inequality.

15.2 Multi-Sector Dynamics

Different economic sectors have different compression ratios and opportunity elasticities. A multi-sector extension would analyze inter-sectoral reallocation effects and the emergence of entirely new sectors. Sectors with high β (e.g., software, content creation) would expand disproportionately, while sectors with low β (e.g., physical manufacturing) would see more modest effects but still experience net work increases through input-output linkages.

15.3 Recursive Self-Improvement

If AI systems can improve their own capabilities (i.e., α depends on W through research output), the

dynamical system acquires an additional feedback channel:

$$\dot{\alpha} = f_{\alpha}(K) + g_{\alpha}(W_{\text{AI-research}}) \quad (61)$$

where $W_{\text{AI-research}} \subset W$ is the work allocated to AI research. The conditions under which this leads to bounded vs. unbounded growth trajectories connect to discussions of artificial general intelligence and are an important open question.

15.4 Information-Theoretic Formulation

There is a natural information-theoretic formulation of the TCP. Define the cognitive channel capacity:

$$C(\alpha) = B_I(\alpha) \cdot \log_2 \left(1 + \frac{S(\alpha)}{N} \right) \quad (62)$$

where $B_I(\alpha)$ is imagination bandwidth, $S(\alpha)$ is AI signal strength (capability), and N is noise (uncertainty, error). This Shannon-like formulation connects the TCP to information theory and provides rigorous bounds on the rate of cognitive work generation.

The opportunity space expansion corresponds to the expansion of the message space when channel capacity increases—analogueous to how increasing internet bandwidth enabled streaming video, a message type that did not exist under narrowband conditions.

15.5 The Yoneda Perspective

From a categorical perspective, the TCP can be understood through the Yoneda lemma. A cognitive task category c is fully characterized by the set of all morphisms into it—all the ways other tasks relate to it. AI does not change the identity of task categories; it changes the *morphisms* (the cost and feasibility of transitions between tasks). By the Yoneda lemma, changing the morphism structure changes the entire representable functor, and thus the entire category of feasible economic activity. This provides a precise categorical expression of why “making things cheaper” does not merely accelerate existing activity but restructures the entire space of possible activities.

16 Conclusion

We have presented the unified theory of the Time Compression Paradox: the counter-intuitive observation that AI, by compressing the time required for cognitive tasks, increases rather than decreases total work. The paradox is resolved by recognizing three interacting mechanisms:

1. **The Jevons Paradox of Intelligence** [Long, 2026a]: Elastic demand for cognitive labor ($\varepsilon > 1$) means that making cognition cheaper increases total cognitive expenditure.
2. **Opportunity Space Expansion** [Long, 2026b]: Reduced task costs render previously infeasible tasks viable, superlinearly expanding the frontier ($\beta > 1$).
3. **Competitive Dynamics** [Long, 2026c]: Market competition forces all agents to exploit the expanded frontier, preventing the time surplus from being consumed as leisure.

These mechanisms do not merely add: they interact through positive feedback loops that are superadditive (Section 7). Jevons drives demand, Opportunity Space provides supply, and Competition forces adoption. The resulting dynamical system (Section 8) admits no finite steady state (Theorem 9.1): total work is monotonically increasing in AI capability (Theorem 3.8).

The formal model predicts a work multiplier that grows as $\rho^{\beta-1}$. For plausible parameters ($\beta \approx 1.7$, $\rho \approx 50\text{--}500$), this implies a 10–100× increase in total cognitive work within a generation. Calibrated simulation (Section 12) projects that the baseline scenario reaches 10× by approximately 2044 and 45× by 2055.

The production constraint shifts from execution time to imagination bandwidth (Section 10). AI is qualitatively different from previous technological revolutions because it amplifies the bottleneck itself—cognitive capacity—rather than a peripheral capability (Section 11).

The deeper insight is that AI does not reduce labor; it increases the speed at which ambition converts into reality. And ambition—the human capacity for wanting more, better, different, deeper—has historically proven to be unbounded.

The AI age will not be an age of leisure. It will be an age of unprecedented cognitive industry, in which the total intellectual output of civilization expands by orders of magnitude. The paradox is that this will feel, to those living through it, like there is *more work than ever*.

More productivity \Rightarrow More possibilities \Rightarrow More projects, \Rightarrow More work

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A Derivation of the Work Multiplier Bound

We provide a detailed derivation of Proposition 5.1.

Let \mathcal{T} denote the universe of conceivable tasks, parametrized by complexity $x \in [0, \infty)$ and value density $v(x)$. The cost of performing task x at AI capability α is:

$$C(x, \alpha) = \frac{x \cdot c_0}{\rho(\alpha)} \quad (63)$$

where c_0 is the base cost rate. A task is feasible if $v(x) \geq \theta \cdot C(x, \alpha)$, i.e.:

$$v(x) \geq \frac{\theta \cdot x \cdot c_0}{\rho(\alpha)} \quad (64)$$

Assume the value density follows a power law: $v(x) = a \cdot x^{-\gamma}$ for $\gamma > 0$ (high-complexity tasks have lower value density per unit of complexity). Then the feasibility condition becomes:

$$a \cdot x^{-\gamma} \geq \frac{\theta \cdot c_0}{\rho(\alpha)} \cdot x \quad (65)$$

Solving for the maximum feasible complexity:

$$x_{\max}(\alpha) = \left(\frac{a \cdot \rho(\alpha)}{\theta \cdot c_0} \right)^{1/(1+\gamma)} \quad (66)$$

The number of feasible tasks (opportunity space) is:

$$|\mathcal{O}(\alpha)| \propto x_{\max}(\alpha)^d = \left(\frac{a \cdot \rho(\alpha)}{\theta \cdot c_0} \right)^{d/(1+\gamma)} \quad (67)$$

where d is the effective dimensionality of the task space. Setting $\beta = d/(1 + \gamma)$, we obtain $|\mathcal{O}(\alpha)| \propto \rho(\alpha)^\beta$.

For $d > 1 + \gamma$ (the task space is high-dimensional relative to the value decay rate), we have $\beta > 1$, confirming Assumption 2.

Total work is the integral of work per task over the d -dimensional task space. Using the radial volume element $x^{d-1} dx$ (since the opportunity space scales as x_{\max}^d , the task space is d -dimensional):

$$W(\alpha) = \int_0^{x_{\max}(\alpha)} \frac{x}{\rho(\alpha)} \cdot c_0 \cdot x^{d-1} dx = \frac{c_0}{\rho(\alpha)} \cdot \frac{x_{\max}(\alpha)^{d+1}}{d+1} \quad (68)$$

Substituting $x_{\max} \propto \rho^{1/(1+\gamma)}$:

$$W(\alpha) \propto \frac{1}{\rho} \cdot \rho^{(d+1)/(1+\gamma)} = \rho^{(d+1)/(1+\gamma)-1} = \rho^{\beta+1/(1+\gamma)-1} \quad (69)$$

where $\beta = d/(1 + \gamma)$. Because $1/(1 + \gamma) > 0$, the exponent exceeds $\beta - 1$: newly feasible tasks at the frontier are more complex than the average pre-existing task, so integrating over the d -dimensional volume yields a *stronger* scaling than the bound $\rho^{\beta-1}$ stated in the main text. The work multiplier bound of Proposition 5.1 is therefore conservative:

$$M_W(\alpha) = \frac{W(\alpha)}{W(0)} \geq \rho(\alpha)^{\beta-1} \quad (70)$$

with equality only in the $d = 1$ case. In general, $M_W \propto \rho^{\beta+1/(1+\gamma)-1} > \rho^{\beta-1}$, which strengthens the TCP thesis.

B Stability Analysis of the Dynamical System

We linearize the system (35)–(39) around a hypothetical fixed point and compute the Jacobian. The eigenvalues of the Jacobian determine local stability.

The Jacobian matrix at a point (O, W, E, K, α) has the structure:

$$J = \begin{pmatrix} \gamma_O \dot{\rho}/\rho & 0 & 0 & 0 & * \\ \lambda_W T_{\text{avg}} & -(\lambda_W + \lambda) & \lambda & 0 & * \\ 0 & \gamma_E & -\gamma_E & 0 & 0 \\ 0 & s\Pi_W & 0 & -\delta & 0 \\ 0 & 0 & 0 & f'_\alpha & 0 \end{pmatrix} \quad (71)$$

The diagonal entries include $\gamma_O \dot{\rho}/\rho > 0$ (positive feedback in opportunity space) and $-\delta < 0$ (capital depreciation). The trace of the Jacobian is:

$$\text{tr}(J) = \gamma_O \frac{\dot{\rho}}{\rho} - (\lambda_W + \lambda) - \gamma_E - \delta \quad (72)$$

For the fixed point to be stable, all eigenvalues must have negative real parts. However, the positive entry $\gamma_O \dot{\rho}/\rho$ in the upper-left, combined with the positive off-diagonal feedback terms ($\lambda_W T_{\text{avg}} > 0$, $s\Pi_W > 0$, $f'_\alpha > 0$), generically produces at least one eigenvalue with positive real part when $\dot{\alpha} > 0$.

More precisely, the submatrix governing (O, W, E) is upper block-triangular with respect to the first row, since the $(1, 2)$ and $(1, 3)$ entries are both zero. Consequently, $\mu_1 = \gamma_O \dot{\rho}/\rho$ is itself an eigenvalue of J_{OWE} . When $\dot{\alpha} > 0$, we have $\dot{\rho} > 0$, so:

$$\mu_1 = \gamma_O \frac{\dot{\rho}}{\rho} > 0 \quad (73)$$

This positive eigenvalue immediately establishes instability: the opportunity space grows exponentially along the corresponding eigenvector whenever AI capability is improving. The remaining eigenvalues of the (W, E) sub-block are the roots of $\mu^2 + (\lambda_W + \lambda + \gamma_E)\mu + (\lambda_W + \lambda)\gamma_E - \lambda\gamma_E = \mu^2 + (\lambda_W + \lambda + \gamma_E)\mu + \lambda_W\gamma_E = 0$, which have negative real parts since all coefficients are positive. Thus the (W, E) sub-system is locally stable, but the positive eigenvalue μ_1 from the opportunity channel dominates, confirming the instability result of Theorem 9.1. The system has no stable interior fixed point as long as $\dot{\alpha} > 0$; all trajectories diverge toward increasing W .

C Simulation Pseudocode

We provide pseudocode for simulating the TCP dynamical system.

Algorithm 1 TCP Dynamical System Simulation

Require: Parameters:

$$\rho_{\text{max}}, k, \alpha_0, \beta, \varepsilon, \gamma_E, \gamma_O, \lambda, s, \delta, \mu, \phi, \eta_\Pi$$

Require: Initial conditions:
 $\alpha(0), O(0), W(0), E(0), K(0)$

Require: Time step Δt , horizon T_{max}

```

1:  $t \leftarrow 0$ 
2: while  $t < T_{\text{max}}$  do
3:   // Compute compression ratio and its derivative
4:    $\sigma \leftarrow 1/(1 + \exp(-k(\alpha - \alpha_0)))$ 
5:    $\rho \leftarrow 1 + (\rho_{\text{max}} - 1) \cdot \sigma$ 
6:    $\dot{\rho} \leftarrow (\rho_{\text{max}} - 1) \cdot k \cdot \sigma \cdot (1 - \sigma) \cdot \dot{\alpha}$ 
7:
8:   // Compute time derivatives
9:    $\dot{\alpha} \leftarrow \mu \cdot K^\phi$ 
10:   $\dot{O} \leftarrow \gamma_O \cdot O \cdot \dot{\rho}/\rho$ 
11:   $T_{\text{avg}} \leftarrow T_0/\rho$ 
12:   $\dot{W}_{\text{target}} \leftarrow O \cdot T_{\text{avg}}$ 
13:   $\dot{W} \leftarrow \lambda_W \cdot (W_{\text{target}} - W) + \lambda \cdot (E - W)$ 
14:   $\dot{E} \leftarrow \gamma_E \cdot (W - E)$ 
15:   $\dot{\Pi} \leftarrow \eta_\Pi \cdot W \cdot (1 - 1/\rho)$ 
16:   $\dot{K} \leftarrow s \cdot \Pi - \delta \cdot K$ 
17:
18:  // Euler integration step
19:   $\alpha \leftarrow \alpha + \dot{\alpha} \cdot \Delta t$ 
20:   $O \leftarrow O + \dot{O} \cdot \Delta t$ 
21:   $W \leftarrow W + \dot{W} \cdot \Delta t$ 
22:   $E \leftarrow E + \dot{E} \cdot \Delta t$ 
23:   $K \leftarrow K + \dot{K} \cdot \Delta t$ 
24:
25:   $M_W \leftarrow W/W(0)$ 
26:  Record  $(t, \alpha, \rho, O, W, E, K, M_W)$ 
27:   $t \leftarrow t + \Delta t$ 
28: end while
29: return trajectory
     $\{(t_i, \alpha_i, \rho_i, O_i, W_i, E_i, K_i, M_{W,i})\}$ 

```

The sigmoid function $\sigma(x) = 1/(1 + e^{-x})$ is used throughout. For production simulations, a fourth-order Runge–Kutta integrator should replace the Euler step. The profit function $\Pi = \eta_\Pi \cdot W \cdot (1 - 1/\rho)$ captures margin improvement from AI: firms capture revenue proportional to W with margin proportional to labor savings $(1 - 1/\rho)$.

The function $f_\alpha(K) = \mu K^\phi$ models AI capability growth with diminishing returns ($\phi = 0.6$). Initial

conditions are: $\alpha(0) = 1$, $O(0) = 1$ (normalized), $W(0) = 1$ (normalized), $E(0) = 1$, $K(0) = 1$. A time step of $\Delta t = 0.1$ years provides adequate numerical stability for the Euler scheme; for higher accuracy, use $\Delta t = 0.01$ with RK4.

The simulation produces the trajectories shown in Figures 5–7 and the sensitivity results in Tables 7–8. All code is available from the YonedaAI Research Collective upon request.